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Random small probability sample matrix used in compressed sensing imaging

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Abstract

This investigation aimed to one common key issue of the two imaging types, the sampling of the template configuration. A first selector framework that employs *MATLAB* to perform attractive features of the small probability matrices in random sampling matrix from Gaussian random matrix was presented. Gaussian random matrix is usually used as random sampling matrix in Single Pixel Camera. The small probability matrices in random sampling matrix can obtain a recovery image of higher accuracy for singe detector imaging system and detector array imaging system.

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Keywords: sampling template; compressed sensing; random matrix selector; flexible electronic device; inspection

1. Introduction

In the single detector imaging system with multi-channel and high flux, such as emerging single pixel camera imaging system [1] and Hadamard Transform Optics [2], tow basic subsystems, which are sampling template and recovery, always exist. The sampling template and recovery of single pixel camera completely dependent on compressed sensing[3] method. Single Pixel Camera has the ability to obtain an image with a single detection element and collecting the number of image that are fewer than image pixels, relying on suitable sampling template and recovery method. It provides a thoroughfare for manufacturing of inspection and measurement, especially for the flexible electronic device. The inspection and measurement of flexible electronic device always need imaging some micro-constructs hidden under the opaque polymer substance, which can be penetrated by THz wave. And therefore, a THz super-resolution imaging system is an important equipment for the inspection and measurement of flexible electronic device, while a suitable sampling template is exactly necessary for this THz super-resolution imaging system. In this paper aiming at the sampling template configuration (sampling matrix), a small probability matrix of binary random matrix and its selector was proposed and demonstrated.

Compressed sensing, which breaks through the classical law of Nyquist sample, aims to recover a sparse image signal x from under sampled indirect measurements $y = Ax \in X^{nxN} (n \le N)$. Then, x is expressed as $x = \Psi s$ with another sparse transform base $\Psi = \{\Psi_1, \Psi_2, ..., \Psi_N\}$. To solve the equations under defined namely NP-hard problem [4, 5], s can be estimated by convex optimization min$\|s\|_1$:

$$\arg\min \|s\|_1 \; s.t. \; y = A\Psi s \qquad [1]$$

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So, the main research contents in compressed sensing are about how to construct a sample matrix A and solve Eq **1** accurately or the most approximately. Candes, Romberg and Tao et al demonstrated that, the sensing matrix $\Theta = A\Psi$ satisfying the conditions of restricted isometric property (RIP) [6] is the guarantee of signal recovery:

$$(1-\delta)\|s\|_2^2 \le \|A\Psi s\|_2^2 \le (1+\delta)\|s\|_2^2$$
[2]

In actuality, it is difficult to use RIP to instruct the design of the sample matrix. The equivalent condition of RIP presented by Baraniuk [7] is that the sample matrix A is irrelevant to sparse transform base Ψ and Gaussian random matrix has little coherence of most of the sparse transform base Ψ in order to satisfy the constraints preferably, which is usually used as sampling matrix in compressed sensing [8,9].

David L. Donoho，Hatef Monajemi et al investigated the concept of deterministic matrix[10]. Now, a series of small probability matrices in random sampling matrix from Gaussian random matrix are studied to realize THz imaging [11] or wide-field super-resolution optical imaging.[12-13] The super-resolution optical imaging is based on the sub-wavelength hole arrays with extraordinary optical transmission (EOT) [14-15].

2. Method

2.1. The inevitability of small probability events

In the classical probability statistics, a small probability event can be defined as an event owning low probability ($p < 0.05$). It is almost impossible to happen in one trial. Assuming that, the probability of a small probability event A is ($\varepsilon < 0.05$), then $\overline{A}(1-\varepsilon)$. B can be defined as the event that A doesn't happen at all in n trials, so the probability of B is $(1-\varepsilon)^n$. Therefore, the probability of $\overline{B}$ occurring at least once in n trails is as follows:

$$P = 1-(1-\varepsilon)^n$$
[3]

$P \to 1$ as $n \to \infty$, thus $\overline{B}$ namely the occurring of small probability event A will always be carried out in the constant independent trails. The transformation theorem of random events [16] presented by Ke-qin Zhao indicates that, if the event A occurred at the K-th test for the first time in the frequency-type random trial E (n is large enough), the event $\overline{A}$ will definitely happen at the $(K+m)-th$. Eq **3** attests that a small probability event of almost impossible occurrence is bound to happen, which aligns with the transformation theorem of random events.

There exists a small probability matrix in the random sampling matrix to obtain a better recovery image.

2.2. The small probability matrix

In our study, the small probability matrix in binary random matrices mainly includes a selected binary matrix by the random matrix selector.

The random matrix selector with *MATLAB* software is designed. The function of the selector is to search a better binary random structure and obtain a preferable recovery by generating various random structures.

2.3. Random matrix selector

A random matrix selector module whose function is to select the random binary matrix using in compressed sensing is designed by *MATLAB*. A random binary matrix is generated , then to select better random binary matrix, and the recovery image is obtained by the gradient projection for sparse reconstruction (GPSR).

In the experiments, the random matrix selector is used to generate thousands of different binary random matrices, and compute the peak signal to noise ratio (PSNR) of the image reconstructed by each sampling matrix. Owning the best performance, the first 18 matrices are shown in Fig 1. Based on the result above, the random matrix selector can be thought as an excellent method to search some preferable properties of sampling matrix.

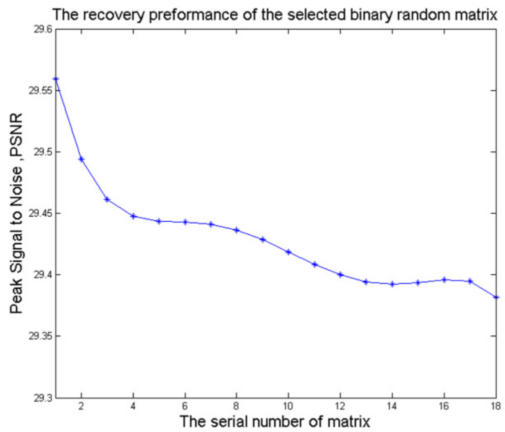

Fig. 1. The recovery performance of the selected binary random matrix.

2.4. Two kinds of recovery algorithms and the sampling mode

Using the GPSR [17] in convex optimization algorithm and the orthogonal matching pursuit (OMP) [18] in greedy algorithm, it can be recovered that the sparse image signal x respectively.

The sampling mode of OMP and GPSR is briefly described in Two imaging mode.

- The imaging mode of GPSR.

The measurements $y = A.*x$ are obtained by means of matrix dot product (multiplying the corresponding element) in the program of *MATLAB*. So each element in the sampling matrix corresponds with each pixel of the original image one to one, which determines the sampling mode of the array detector in the super-resolution imaging experiment. The measurements y can be obtained by sampling one frame or several frames of image. The iterative process of GPSR is as follows:

1 The initial feasible point $x_1 \in R^n$, the iterative threshold $\varepsilon \geq 0$ and $d_1 = -g_1$, $k = 1$ are given. If $\|g_1\| \leq \varepsilon$, the iterative process is stopped;

2 The searching step α_k meets Armijo line search;

3 $x_{k+1} = x_k + \alpha_k d_k$, $g_{k+1} = g(x_{k+1})$, if $\|g_{k+1}\| \leq \varepsilon$, the iterative process is stopped;

4 The parameters $\beta_{k+1} = -\|g_{k+1}\|^2 / d_k^T g_k$ and $d_{k+1} = -\theta_{k+1} g_{k+1} + \beta_{k+1} d_k$ are updated;

5 $k = k+1$, return to step 2.

In the iterative process, $g_k = \nabla f(x_k)$ is the Gradient of x_k, $\alpha_k \geq 0$ is the searching step factor and $\theta_k = 1 - \frac{g_k^T d_{k-1}}{d_{k-1}^T g_{k-1}}$.

- The imaging mode of OMP.

The measurements $y = A \times x$ are obtained by the way of standard matrix multiplication in the program of *MATLAB*. For the convenience of computing, the two-dimensional original image x is expanded into a one-dimensional column vector signal $\vec{x}$. The sampling matrix can be expressed as the set of row vector $A = (\alpha_1, \alpha_2, ..., \alpha_i)'$. Then, the measurements can be expressed as $y_i = [\alpha_i, \vec{x}]$ where the $[\alpha_i, \vec{x}]$ is the inner product of α_i and $\vec{x}$. The process above determines the sampling mode of the single detector in the imaging (or super-resolution imaging) experiment. The measurements y can be obtained only by the cyclic moving sampling with one imaging template based on digital micro-mirror device (DMD) (or based on Hadamard Transform Optics).

2.5. Image quality assessment

- Some parameters to evaluate the image quality.

When the recovery image is got, some parameters to evaluate the image quality and identify the performance of reconstruction algorithm or the imaging equipment are often needed. There exist some image evaluation methods, such as the mean square error (MSE), the relative error (RE) and the PSNR

$$mse = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|x(i,j) - \hat{x}(i,j)\|^2 \quad [4]$$

$$re = \|x - \hat{x}\|_2 / \|x\|_2 \quad [5]$$

$$psnr = 10 \log_{10}(\max{}^2 / mse) \quad [6]$$

In which, m and n represent the number of rows and columns of gray matrix. x, $\hat{x}$ denote the original image and the recovery image respectively. The parameter *max* expresses the grayscale of image (0~255).The situation that the evaluation results are inconsistent with the person's subjective perception exists in the methods above due to not considering the intuitionistic view of human. So the signal noise ratio (SNR) of image is necessary to select the reconstruction algorithms. The ratio of maximum and minimum in the image local variance is used as the SNR of image:

$$SNR \approx \sigma_{\max} / \sigma_{\min} \quad [7]$$

The local variance of measurements y in the position (i, j) is defined as:

$$\sigma(i,j) = \beta \sum_{k=-P}^{P} \sum_{l=-Q}^{Q} [y(i+k, j+l) - \mu_y(i,j)]^2 \quad [8]$$

In which, μ_y denoting the local average is computed as follows:

$$\mu_y = \beta \sum_{k=-P}^{P} \sum_{l=-Q}^{Q} y(i+k, j+l) \quad [9]$$

$$\beta = \frac{1}{(2P+1)(2Q+1)} \quad [10]$$

In which, the window size $P = Q = 1$ (3×3) or $P = Q = 2$ (5×5) is used to calculate the local variance. The approach regarding the smallest area of the local variance as the flat area is reasonable. The SNR of some smooth blur images is too large to reflect the nature of bad images resulting in the deviation of quality evaluation. So the concept of SNR improvement is presented to make up for this deficiency. It is computed as follows:

$$M_{dB} = 10 \cdot \lg \frac{\sum_{i,j \in D_x} (y(i,j) - x(i,j))^2}{\sum_{i,j \in D_x} (\hat{x}(i,j) - x(i,j))^2} \quad [11]$$

In which, $y, x, \hat{x}$ means the measurements, the original image and the recovery image respectively. D_x is the Region of x.

In comprehensive comparison, RE means the two-norm radio of absolute residual and original image. It is proportional to the absolute residuals in the situation of fixed original image. Therefore, it can be believed that RE is the most valuable parameter corresponding with the intuitionistic view of human. As is shown in Eq 4, MSE denotes the average value of the residual sum of squares whose variation is consistent with RE. So it is usually used as the evaluation parameter. The PSNR which is widely used to evaluate the image quality is expressed as the ratio of the maximum signal and the background noise. However, it is slightly different with the intuitionistic view of human due to that the human visual sensitivity to error is not absolute. The Eq 6 shows that the trend of PSNR and RE is opposite. So the PSNR can be used to assist RE to judge the merits of image quality. As we know, the M_{dB} is similar as the SNR boost $\sqrt{n}$ obtained by Hadamard matrix in classical Hadamard Transform Optical imaging method. In Eq 7, the SNR of image is used to measure the amount of the image noise without the original image for reference. The SNR of some smooth blur images is too large to reflect the nature of bad images resulting in the deviation of quality evaluation. So the SNR of image should only be used as a general reference.

- Residual analysis.

To further explicit the worth of parameters to evaluate the image quality, the residual of $\hat{x}$ is studied using different sampling matrices and different reconstruction algorithms. The normalized residual is regarded as the absolute residual. The two-dimensional image $x(i,j)\ i,j\in(1,256)$ is expanded as the one-dimensional signal $x(1,256(i-1)+j)$. Then, the image pixel location $256(i-1)+j$ is treated as the abscissa and the absolute residual is treated as the ordinate. It can be plot that a set of absolute residual distribution. The statistical results are shown in Table 1 regarding the absolute residual of 7.6% as the baseline. There is a little bad points among the total residuals of 65536 (256×256).

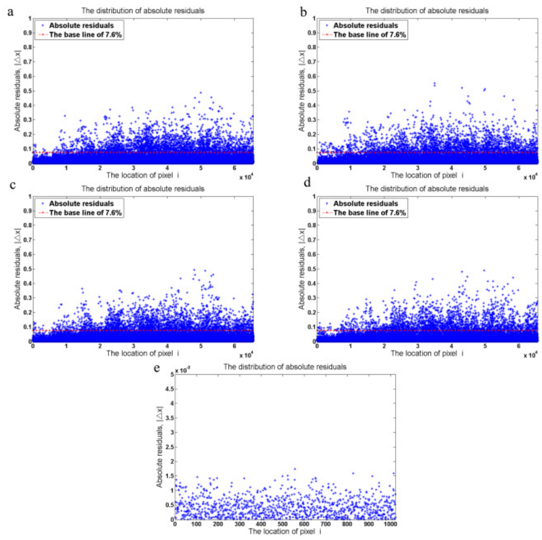

Fig. 2. The absolute residual distributions of different sampling matrices. (*a*) the distribution of Selected binary random matrix (PSNR29.31); (*b*) the distribution of Selected binary matrix (PSNR29.46); (*c*) the distribution of Selected binary matrix(PSNR29.49); (*d*) the distribution of Selected binary matrix(PSNR29.57); (e) represent the absolute residual distributions under OMP.

In Fig 2, the red line is the baseline of 7.6%. The first five images represent the absolute residual distributions of the binary random matrix and selected binary matrix. This coincides with the relative errors baseline $(RE < 7.6\%)$. The sixth image denotes the absolute residual distributions of a array under OMP. It is obviously that the residual value is very small and almost negligible.

Table 1. The statistics about the number of absolute residual (> 7.6%).

The style of matrix	The number of residual	The rate
Selected binary matrix PSNR29.31	1496	2.28%
Selected binary matrix PSNR29. 46	1484	2.26%
Selected binary matrix PSNR29. 49	1476	2.25%
Selected binary matrix PSNR29. 57	1428	2.17%

3. Results

- Study in the recovery performance of small probability matrix with GPSR.

Using the selected binary random matrix as the sampling matrix in GPSR, the MSE, the RE, the PSNR, the SNR of image and the SNR boost (M_{dB}) are calculated. Results are shown in Table 2. As benchmark for the binary random matrix (RE < 7.6%), these matrices above are desirable. The four image obtained by using binary random matrix， the selected binary random matrix, are shown in Fig **3**. They all have an ideal quality of recovery from the intuitionistic view of human.

- Study in the recovery performance of small probability matrix with OMP.

Repeat the steps above. As is known, the nature of OMP is using single detector to obtain the recovery image. The region of template corresponding with single detector is the approximate square. When a DMD is used as sampling set, not only the sampling mode can be altered but also can be used in reflecting imaging mode. The selected binary random matrix as the sampling matrix in OMP with DMD can obtained a better recovery image.

Table 2. The recovery performance of selected random matrix under GPSR.

The style of matrix	MSE	RE	PSNR(dB)	SNR	M_{dB}
Selected random matrix (a)	0.0012	0.0732	29.3113	38.1066	20.4797
Selected random matrix (b)	0.0012	0.0721	29.4626	37.8023	20.3172
Selected random matrix (c)	0.0011	0.0719	29.4951	37.6137	20.3075
Selected random matrix (d)	0.0011	0.0702	29.5702	37.5325	20.2986

Fig. 3. The four Lena image obtained by using the selected binary random matrix according to the sequence in table 2.

4. Conclusion

It is well demonstrated that a concept of small probability matrix from the compressed sensing sampling matrix with the inevitability theory of small probability event. The concept of small probability binary random matrix and the implementation of its selector can fundamentally alter the sample template randomness of the Single Pixel Camera. It makes good use of the characteristic of Gaussian random matrix which is hardly correlated with any sparse signal and also not affected by the randomness of random matrix. Thus, the sample template can not only meet the criteria of RIP to recovery images by compressed sensing method, but also ensure the high PSNR of the recovery images.

For array detector, the selected binary random matrix can be used as the sample template of super resolution imaging by studying the recovery performance of them. The SNR boost among them are higher than 20, while it is preferable of the recovery performance using selected binary random matrix designed reflecting imaging template for single detector under the DMD. These matrices provide an important reference for achieving the wide-field super diffraction limit resolution optical and THz imaging with compressed sensing methods.

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